NATIONAL EDUCATION POLICY-2020

Common Minimum Syllabus for all

Uttarakhand State Universities and Colleges

for Five Years of Higher Education

PROPOSED STRUCTURE

OF POST GRADUATE - MATHEMATICS

SYLLABUS

2023

Curriculum Design Committee, Uttarakhand

S. No.	Name & Designation	
1.	Prof. N.K. Joshi Vice-Chancellor, Sridev Suman Uttarakhand University, New Tehri	Chairman
2.	Vice-Chancellor, Kumaun University, Nainital	Member
3.	Prof. Jagat Singh Bisht Vice-Chancellor, Soban Singh Jeena University Almora	Member
4.	Prof. Surekha Dangwal Vice-Chancellor, Doon University, Dehradun	Member
5.	Prof. O. P. S. Negi Vice-Chancellor, Uttarakhand Open University, Haldwani	Member
6.	Prof. M.S.M. Rawat Advisor, Rashtriya Uchchatar Shiksha Abhiyan, Uttarakhand	Member
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Curriculum Design / Syllabus Framing Committee

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1	Prof. Anita Tomar Professor & Head Dept of Mathematics Sridev Suman Uttarakhand University Pt. LMS Campus Rishikesh	Expert
2	Prof. Dipa Sharma Professor Dept of Mathematics Sridev Suman Uttarakhand University Pt. LMS Campus Rishikesh	Expert
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4	Dr. Dhirendra Singh Assistant Professor Dept of Mathematics Sridev Suman Uttarakhand University Pt. LMS Campus Rishikesh	Expert

Department of Mathematics

B.Sc fourth and fifth year (VII, VIII, IX & X Sem.)

degree with research and

P.G. Syllabus of Mathematics

Year	Sem.	Paper Title	Theory/ Practical	CREDIT (L+P+T)
UG Fourth Year / PG First Year	**UG VII / PG I Sem.**	Real Analysis	Theory	**5 (4+0+2)**
		Topology	Theory	**5 (4+0+2)**
		Differential Geometry and Tensor Calculus	Theory	**5 (4+0+2)**
		Mathematical Methods/Transformations	Theory	**5 (4+0+2)**
		Research Project		**4 (4+0+0)**
	UG VIII / PG II Sem	Complex Analysis	Theory	**5 (4+0+2)**
		Abstract Algebra	Theory	**5 (4+0+2)**
		Differential Equations	Theory	**5 (4+0+2)**
		Discrete Mathematics	Theory	**5 (4+0+2)**
		Research Project		**4 (4+0+0)**
UG Fifth Year / PG Second Year	**UG IX / PG III Sem**	Linear Algebra	Theory	**5 (4+0+2)**
		Dynamics of Rigid Bodies	Theory	**5 (4+0+2)**
		Number Theory	Theory	**5 (4+0+2)**
		Measure Theory	Theory	**5 (4+0+2)**
		Research Project		**4 (4+0+0)**
	UG X / PG IV Sem	Functional Analysis	Theory	**5 (4+0+2)**
		Special Functions	Theory	**5 (4+0+2)**
		Calculus of Variations And Integral Equations	Theory	**5 (4+0+2)**
		Operation Research	Theory	**5 (4+0+2)**
		Research Project		**4 (4+0+0)**

Detailed Syllabus of Courses

Course1: RealAnalysis

Unit 1. Metric spaces: definition and examples,Cauchy sequences, completeness, open set, interior of a set, limit points, closure of a set, closed sets, continuous mappings, uniform continuity, homeomorphism, compactness and connectedness.

Unit 2. Functions of several variables: Concept of functions of two variables, Simultaneous and iterated limits in functions of two variables, Partial derivatives: Definition, Existence and continuity, Interchange of order of differentiation, Directional derivatives.

Unit3. Composite functions, Continuity of function of two variables, differentiability of functions of two variables, Taylor's Theorem.

Unit4. Linear transformation, Vector Valued functions, Differentiation of vector valued functions, inverse function theorem, implicit function theorem.

Books Recommended:

- *S.C.MalikandSavitaArora:MathematicalAnalysis,NewAgeInternational.*
- *Satish Shirali and Harikishan L. Vasudeva, Metric Spaces*
- *G.F.Simmons:IntroductiontoTopologyandModernAnalysis,TataMcGrawHill.*
- *W.Rudin:PrinciplesofMathematicalAnalysis(3rd edition),TataMcGrawHill Kgakusha, International Student Edition, 1976.*
- *T.M.Apostol:MathematicalAnalysis,NarosaPublishingHouse,NewDelhi,1985.*

Course 2: Topology

Unit 1. Basic concepts in Topology: Topology on a set, a topological space with examples, topologies on the real number system.

Unit 2. Neighborhood of a point/set,Openand closed sets, interior, boundary, closure, limit point, Derived sets ofa set, Base and sub-base of a topology, Separable Spaces,First and Second Countable spaces,

Unit 3. Continuous map, open and closed maps, homeomorphisms, Topological invariants, Pasting Lemma, Subspaces, product spaces, quotient space.

Unit 4. Compactness, Compact spaces, Compactness of a metric space, Connectedness, connected space, path-wise connected space, components. Separation axioms: , regular, completely regular and Normal space.

Books Recommended:

- *J.R.Munkres:Topology:NarosaPublishingHouse.*
- *Shaum'soutlinesseries:TataMcGrawHill.*
- *K.D.Joshi:IntroductiontoGeneralTopology,WileyEastern,1983.*
- *G.F.Simmons:IntroductiontoTopologyandModernAnalysis,McGrawHill,1963.*
- *M.D.Raisinghania&R.S.Aggarwal:Topology,S.Chand&Co.*

Course 3: Differential Geometry and Tensor Calculus

Unit 1. Curve in space, parameterized curves, regular curves, helices, arc length, re-parameterization (byarc length),Tangent, principalnormal, binormal,osculatingplane, normal plane, rectifying plane, curvature torsion of smooth curves, Frenet- Serret formulae, Frenet approximation of space curve.

Unit2. Order of contact, osculating circle, osculating sphere, Spherical indicatries, involutes and evolutes, Bertrand Curves, intrinsic equations of space curves, isometries ofR^3, Fundamental theorem of space curves, surfaces in R^3.

Unit3. Curvature of curves on surfaces, normal curvature, principal curvatures, geometric interpretation of principal curvatures, Euler theorem, mean curvature, lines of curvature, Rodrigue's formula, umbilical points, minimal surfaces, definition and examples, Gaussian curvature, intrinsic formulae for the Gaussian curvature, isometries of surfaces,.

Unit4. n-dimensional real vector space, contravariant vectors, dual vector space, Covariant vectors, tensor product, second order tensors, tensors of type (r, s), symmetry and skew symmetry of tensors, fundamental algebraic operations: Addition, multiplication, contraction and inner product. Quotient law of tensors.

Books Recommended:

- *C.E.Weatherburn:RiemannianGeometryandTensorCalculus.*
- *AndrewPressley:ElementaryDifferentialGeometry,Springer(UndergraduateMathematics Series), 2001.*
- *J.A.Thorpe:ElementaryTopicsinDifferentialGeometry,Springer(UndergraduateTextsin Mathematics), 1979.*
- *D.Somasundaram:DifferentialGeometry,AFirstCourse,NarosaPublishingHouse,New Delhi, 2005.*
- *T.J.Willmore:AnIntroductionToDifferentialGeometry,OxfordUniversityPress.*

Course 4: Mathematical Methods/Transformations

Unit 1. Laplace Transforms: Definition, Existence theorem, Linearity property, Laplace transforms of elementary functions, Heaviside Step and Dirac Delta Functions, First Shifting Theorem, Second Shifting Theorem, Initial-Value Theorem, Final-Value Theorem, The Laplace Transform of derivatives, integrals and Periodic functions.

Unit2. Inverse Laplace transforms: Inverse Laplace transforms of simple functions, Inverse Laplace transforms using partial fractions, Convolution, Solutions of differential and integro-differential equations using Laplace transforms. Dirichlet's condition.

Unit3. Fourier Transforms: Fourier Complex Transforms, Fourier sine and cosine transforms, Properties of FourierTransforms, Inverse Fourier transforms.

Unit4. Applications of Fourier transform to simple one dimensional heat transfer equations, wave equations and Laplace equations, Z Transform and its application to solve difference equations.

Books Recommended:

- *Murry R. Spiegal: Laplace Transform (SCHAUM Outline Series), McGraw-Hill.*
- *J. F. James: A student's guide to Fourier transforms, Cambridge University.*
- *Ronald N. Bracewell: The Fourier transforms and its applications, Mcgraw Hill.*
- *J. H. Davis: Methods of Applied Mathematics with a MATLAB Overview, Birkhäuser, Inc.,Boston, MA,* 2004.

Course 5: Complex Analysis

Unit 1. Derivatives of an analytic function, Higher order derivatives, Cauchy's integral formula, Cauchy inequality and Liouville's theorem, Maximum modulus principle, Schwarz lemma, The open mapping theorem.

Unit 2. Taylor's series, Laurent Series, Classification of Singularities, Calculus of Residues, Cauchy's Residue theorem, evaluation of integral.

Unit3. Entire functions, Hadmard's three circle theorem, Meromorphicfunctions,the argument principle, Rouche's theorem **,** The fundamental theorem of algebra.

Unit4. Conformal mappings, Power series representation of analytic functions, Analytic functions as mappings, Riemann sphere, Linear transformations, Mobius transformation, Cross ratios, Mobius transformation on circles.

Books Recommended:

- *L.V.Ahlforse:ComplexAnalysis,TataMcGrawHill.*
- *J.B.Conway:FunctionsofoneComplexvariable,Springer-Verlag,1980.*
- *D.Sarason:ComplexFunctionTheory,HindustanBookAgency,Delhi,1994.*
- *B.Choudhary:ElementsofComplexAnalysis,WileyEasternLtd.,NewDelhi,1993*

Course 6: Abstract Algebra

Unit 1. Introductions of group, Relation of conjugacy, conjugate class of a group, class

equation, Cayley's theorem, Sylow's theorem.

Unit 2. Normal and subnormal series, composition series, Jordan Holdertheorem, chain conditions, Commutators. Solvable groups, solvability of subgroups and factor groups, Nilpotent groups and their equivalent characterizations.

Unit 3. Rings, ideals, prime and maximal ideals, quotient rings. Factorisation theory in commutative domains.Prime and irreducible elements, Euclidean Domains. Principal Ideal Domain. Divisor chain condition. Unique Factorisation Domains, examples and counter examples. Polynomial rings over domains. Eisenstein's irreducibility criterion .Unique factorisation in polynomial rings over U.F.D.s.

Unit 4. Fields,finitefields,fieldextensions,Galoisextensions.

BooksRecommended:

- *J.Gallian:AbstractAlgebra,NarosaPublication.*
- *N.Jacobson:BasicAlgebra, Vol.1,HindustanPublishingCo.,NewDelhi.*
- *M.Artin:Algebra,PrenticeHallofIndia.*
- *RamjiLal:FundamentalsinAbstractAlgebra,ChakraPrakashan,Allahabad,1985.*
- *I.N.Herstein:TopicsinAlgebra,WileyEasternLtd.,N.D.,1975.*
- *D.S.DummitandR.M.Foote:AbstractAlgebra,JohnWiley,N. Y.*

Course 7: Differential Equations

Unit 1. Existence and uniqueness of solutions of initial value problems for first order ordinary differential equations, singular solutions of first order ODEs, system of first order ODEs., General theory of homogeneous and non-homogeneous linear ODEs, variation of parameters, Sturm-Liouville boundary value problem, Green's function.

Unit2. FormationofP.D.E.'s.FirstorderP.D.E.'s,Classificationoffirstorder,P.D.E. 's,Complete, general and singular integrals, Lagrange's or quasi-linear equations, Integral surfaces through a given curve. Orthogonal surfaces to a given system of surfaces, Characteristic curves.

Unit 3. Pfaffiandifferential equations, Compatiblesystems, Charpit'smethod, Jacobi's Method.Cauchy problem for first order PDEs.

Unit 4. Linear equations with constant coefficients, Reduction to canonical forms, Classification of second order P.D.E.s.Generalsolution of higher order PDEs with constant coefficients.

Books Recommended:

- *M.D.Raisinghania,AdvancedDifferentialEquations,S.Chand2016.*
- *D.P.ChoudharyandH.I.Freedman:ACourseinOrdinaryDifferentialEquations,Narosa Publishing House, New Delhi,2002.*
- *E.A.Coddington: An IntroductiontoOrdinaryDifferentialEquations,PrenticeHallofIndia, New Delhi,1968.*
- *T.Amaranath:AnElementaryCourseinPartialDifferentialEquations,NarosaPublishing House, New Delhi, 2005.*
- *ErwinKreyszig:AdvancedEngineeringMathematics,JohnWiley&SonInc.,NewYork, 1999.*

Course 8: Discrete Mathematics

Unit 1. Principle of mathematical induction, partially ordered sets, Lattices: Lattices as partially ordered sets, Their Properties, Lattices and algebraic systems. Principle of duality, Sub lattices, Complete, Complemented and Distributive lattices.

Unit 2. Boolean algebra, Boolean functions, Boolean expressions, Applications to switching circuits.

Unit 3. Elements of graph theory: Basic terminology, Path sand circuits, Eulerian and Hamiltonian graphs, planar graphs, Directed graphs.

Unit 4. Trees: Rooted trees, path lengths, spanning trees, minimum spanning trees.

Books Recommended:

- *C. L. Liu: Elements of discrete mathematics, Tata McGrawHill Education,2008.*
- *Ram Babu: Discrete Mathematics, Pearson Edition India, 2011.*
- *Lipschutz: Discrete Mathematics,TataMcGrawHill, 2011.*

Course 9: Linear Algebra

Unit 1. A brief review of vector space, Inner products, Orthogonality, Best approximations, Projections, Cauchy-Schwartz inequality.

Unit 2. Adjoint of a linear transformation, Self adjointtransformations,Unitaryoperators. Normal operators: Definition and properties and Spectral theorem.

Unit 3. Eigen vectors and eigen values of a linear operator,Minimal polynomial of a linear operator and its relations to characteristic polynomial, Cayley-Hamilton theorem.

Unit 4. Bilinear forms, Symmetric and skew-symmetric bilinear forms, Groups preserving bilinear forms.

BooksRecommended:

- *Hadley:LinearAlgebra.*
- *HoffmanandKunz:LinearAlgebra,PrenticeHallofIndia,NewDelhi,1972.*
- *H.Helson:LinearAlgebra,HindustanBookAgency,NewDelhi,1994.*
- *K.B.Dutta:MatrixandLinearAlgebra,PrenticeHallofIndia.*

Course 10: Dynamics of rigid Bodies

Unit1.D'Alembert'sprinciple, Motion about a fixed axis(Finite and Impulsive forces).

Unit 2. Motion in two dimensions under Finite and Impulsive forces, Principle of conservation of momentum and energy.

Unit3. Lagrange'sequations in generalized co-ordinates.

Unit4. Hamilton'sprinciple, principle of least action, Euler's geometrical and dynamicalequations.

Books Recommended:

- *S.L.Loney:Dynamicsofrigidbodies.*
- *BhuDevSharma:DynamicsofRigidBodies,KedarnathRamnathSons,1984.*
- *M. Ray &Harswarup Sharma: A text book of Dynamics of Rigid Body, Students' Friends&Co., Agra-2, 1971.*
- *A.S.Ramsey:Dynamics–PartII.*
- *H.Goldstein:ClassicalMechanics,Narosa,1990.*

Course 11: Number Theory

Unit 1. Mathematical Induction, Binomial Theorem, Division Algorithm, Greatest Common Divisor, Euclidean Algorithm, Diophantine Equation: , Prime Numbers, Unique Factorization Theorem.

Unit 2. Congruence, Residue, Fermat's Theorem, Wilson Theorem, Number Theoritic Function: and , Euler's Phi Function, Euler's Theorem.

Unit 3. Primitive Roots, Legendre Symbol, Quadratic Reciprocity Law, Prefect Numbers, Fermat Numbers.

Unit 4. Diophantine Equation of the form , Fibonacci Sequence, Continued Fraction, Farey Fraction.

Books Recommended:

- *G.H. Hardy and E.M. Wright: Introduction to the theory of numbers, Oxford University Press, 4th Edition.*
- *D.M. Burton: Elementary Number Theory, 6th Edition, Tata McGraw Hill.*
- *Thomas Koshy: Elementary Number Theory with Applications, Academic Press, 2ndEdition.*
- *Kenneth H. Rosen: Elementary Number Theory and its Applications, Addison Wesley Publishing Company, 1986.*

Course 12: Measure Theory

Unit1. Countable sets, uncountable sets, relation between the cardinality of a nonempty set and the cardinality of its power set; Boolean ring, σ-ring, Boolean algebra and σ-algebra of sets, Set function.

Unit2. Introduction, Outer measure, Measurable sets and Lebesgue measure, Example of non-measurable sets, Measurable functions.

Unit3. The Riemann integral, The Lebesgue integral of a bounded function over a set of finite measure, The integral of nonnegative functions. The general Lebesgue integral on , Convergence in measure.

Unit 4. Differentiation of monotone functions, Functions of bounded variation, Differentiation of an integral, Absolute continuity, Convex functions.

Books Recommended:

- *P.K.Jain:MeasureTheory,NewAgeInternational.*
- *P.R.Halmons:MeasureTheory,GrandTextMathematics,14Springer,1994.*
- *E.HewitandK.Stromberg:RealandAbstractAnalysis,Springer,1975.*
- *K.R.Parthasarathy:IntroductiontoProbabilityandMeasure,TRIM33,Hindustan Book Agency, New Delhi, 2005.*
- *I.K.Rana:AnIntroductiontoMeasureandIntegration,(SecondEdition),NarosaPublishing House, New Delhi, 2005.*

Course13: Functional Analysis

Unit 1. Metric convergence of sequences, Normed spaces, Banach Space, Properties of Normed spaces, Finite dimensional normed spaces and subspaces; Compactness and finite dimension, linear operators, Bounded and continuous linear operators; Linear functional; linear operatorsand functional on finite dimensional spaces, Normed spaces of operators, Dual space.

Unit 2. Inner product space; Hilbert space; Properties of Inner product spaces, Orthogonal complements and direct sums, Orthonormal sets and sequences; Hilbertadjoint operators, Self- Adjoint, Unitary and normal operators.

Unit 3. Fundamental Theorems of Normed and Banach Space: Zorn's Lemma, Hahn Banach Theorem, Hahn Banach Theorem for complex vectorspaces and normed spaces, Adjoint operators, Uniform boundedness theorem, strong and weak convergence, convergence of sequences of operators and functional, Open mapping theorem and closed graph theorem.

Unit 4. Fixed point, Contraction mapping, Banach contraction principle, Applications of Banach's theorem to linear, differentialand integral equations.

BooksRecommended:

- *ErwinKreyszig:IntroductoryFunctionalAnalysis,WileyIndiaedition.*
- *G.F.Simmons:IntroductiontoTopologyandModernAnalysis,McGrawHill,1963.*
- *A.E.Taylor:IntroductiontoFunctionalAnalysis,JohnWiley,1958.*
- *R. E. Edwards: Functional Analysis, Holt Rinehart and Winston, 1965.*

Course 14: Special Functions

Unit 1. Preliminaries, Gamma function and related functions, Gauss multiplication theorem, the hypergeometric differential equation, Gauss hypergeometric function.

Unit 2. Integral representation of hypergeometric function, Evaluation of hypergeometric function, the confluent hypergeometric differential equation, Confluent hypergeometric function.

Unit 3. Bessel's equation, solution of Bessel's equation, Bessel's functions Recurrence Formulae, Equations reducible to Bessel's equation, orthogonality of Bessel's Functions, A generating function for Basic properties.

Unit 4. Legendre's equation, Legendre's polynomial , Legendre's function of the second kind General solution of Legendre's equation, Rodrigue's formula, Legendre polynomials, A generating function of Legendre's polynomial , Orthogonality of Legendre polynomials, Recurrence formulae for

Books Recommended:

- *E. D. Rainville: Special functions.*
- *Nirvikar Saran: Special Functions.*
- *W.W.Bell:SpecialFunctionforScientistsandEngineers,Deverpublications, 2002,*
- *U. P. Singh: Special Function & Their Application, WISDOM PRESS, 2012.*

Course 15: Calculus of Variations and Integral equations

Unit 1. Variation and its properties, Linear functional, The fundamental lemma of the calculus of variations, Euler-Lagrange equations, Variational problems for functional involving several dependent variables, Functional depends on higher order derivative, Functional dependent on the functions of several independent variables, Variational problems in parametric form, Isometric problems.

Unit 2. Sufficient conditions for extremum**:** Proper field, Central field, Extremal field, Jacobi condition, Weierstrass function, Legendre condition.
Unit 3. Linear integral equation: Definition and classification with various examples, Conversion of IVP and BVP to an integral equations and vice-versa, Eigen value and eigen functions, Solution of homogeneous and general Fredholm integral equations of second kind with separable and degenerate kernels.
Unit 4. Solution of Fredholm and Volterra integral equations of second kind by method of successive approximations, Resolvent kernels and its results.

Books Recommended:

- *L. Elsgolts: Differential Equations and Calculus of Variations, Mir Publishers, 1970.*
- *A.S.Gupta:CalculusofVariations,PrenticeHallofIndia,NewDelhi,1999.*
- *J.H. Davis: Methods of Applied Mathematics with a MATLAB Overview, Birkhäuser, Inc., Boston, MA, 2004.*
- *L. G. Chambers: Integral Equations A short Course, Int. Text Book company Ltd. 1976.*
- *Abdul J Jerry: Introduction to Integral Equations with Applications, Marshal and Dekkar.*
- *Naveen Kumar: An Elementary Course on Variational Problems in Calculus, Narosa, 2004.*

Course 16: Operation Research

Unit 1. Development of OR, Definition, characteristics, scope, objectives and limitations of OR, Formulation of LPP, Graphical Method to solve LPP, General LPP, Canonical and Standard forms, Properties of Solutions and Theory of Simplex method, Big M Method and Two phase simplex method, Degeneracy in LPP. Duality in LPP, Duality and simplex method, Dual simplex method, Revised simplex method and bounded variable problems.
Unit 2. Pure and Mixed integer programming, GomoryallIPPmethod,Fractionalcutmethod,LpFormulationofTP,TransportationTable,Finding initial basic feasible solution,Test of optimality,Degeneracy,MODImethod,SteppingStone method, Solutions of Assignment problems, Hungerian method, Duality in assignment problem.
Unit 3. Changes in Objective Function Coefficient, Changes in constants, Changes in coefficients of decision variables in constraints, Structural changes.
Unit 4. Network flow problem, minimal spanning tree problem, shortest route problem, maximal flow problem, minimum cost flow problems, critical path analysis, PERT and CPM, Formulation of NLPP, general NLPP, constrained optimization with equality and unequality constraints.

BooksRecommended:

1.H.A.Taha:OperationsResearch,AnIntroduction,Pearson.

- *KantiSwarup,PKGupta,Manmohan:OperationsResearch,SultanChand&Sons,New Delhi.*
- *S.S.Rao:OptimizationTheoryandApplicationsWileyEastern.*
- *F. S. Hiller and G. J. Leiberman: Introduction to Operation Research Edition), McGrawHill International Edition, 1995*